

Exercise 41

Use logarithmic differentiation to find the derivative of the function.

$$y = \sqrt{\frac{x-1}{x^4+1}}$$

Solution

Take the natural logarithm of both sides and use the properties of logarithms to simplify the right side.

$$\begin{aligned}\ln y &= \ln \sqrt{\frac{x-1}{x^4+1}} \\ &= \ln \left(\frac{x-1}{x^4+1} \right)^{1/2} \\ &= \frac{1}{2} \ln \frac{x-1}{x^4+1} \\ &= \frac{1}{2} [\ln(x-1) - \ln(x^4+1)]\end{aligned}$$

Differentiate both sides with respect to x .

$$\begin{aligned}\frac{d}{dx}(\ln y) &= \frac{d}{dx} \left\{ \frac{1}{2} [\ln(x-1) - \ln(x^4+1)] \right\} \\ \frac{1}{y} \cdot \frac{d}{dx}(y) &= \frac{1}{2} \left[\frac{1}{x-1} \cdot \frac{d}{dx}(x-1) - \frac{1}{x^4+1} \cdot \frac{d}{dx}(x^4+1) \right] \\ \frac{1}{y} \cdot \frac{dy}{dx} &= \frac{1}{2} \left[\frac{1}{x-1} \cdot (1) - \frac{1}{x^4+1} \cdot (4x^3) \right] \\ \frac{1}{y} \frac{dy}{dx} &= \frac{1}{2} \left(\frac{1}{x-1} - \frac{4x^3}{x^4+1} \right) \\ \frac{dy}{dx} &= \frac{y}{2} \left[\frac{1(x^4+1) - 4x^3(x-1)}{(x-1)(x^4+1)} \right] \\ &= \frac{1}{2} \sqrt{\frac{x-1}{x^4+1}} \left[\frac{-3x^4 + 4x^3 + 1}{(x-1)(x^4+1)} \right] \\ &= \frac{-3x^4 + 4x^3 + 1}{2(x^4+1)\sqrt{(x^4+1)(x-1)}}\end{aligned}$$